Kites, pseudo-MV algebras and ℓ-groups

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A kite is a certain construction with origins in work of Jipsen and Montagna, later developed by Dvurecenskij and TK, and later yet by Botur and TK. In particular, it yields a categorical equivalence between perfect pseudo-MV algebras and lattice-ordered groups with a distinguished automorphism. Combining several well-known facts about ℓ-groups, residuated lattices and pseudo-MV algebras, this result can be presented as a pure ℓ-group embedding result. Iterating the embedding we can construct an embedding of an arbitrary ℓ-group into an ℓ-group with no nontrivial outer automorphisms. The result is not new, it was known as early as 1973 to McCleary, under the assumption of GCH. In 2000, Droste and Shelah, eliminated the need for GCH. But both McCleary and Droste-Shelah results are in fact about Holland representations: they show that any ℓ-group $Aut(C)$ for a chain C can be embedded into an ℓ-group $Aut(D)$ for a chain D, such that $Inn(Aut(D)) = Aut(Aut(D))$. We show less, namely, that any ℓ-group $\mathbf{G}$ can be embedded into an ℓ-group $\mathbf{H}$ such that $Inn(\mathbf{H}) = Aut(\mathbf{H})$, but we use elementary techniques that we discovered via working with kites and pseudo-MV algebras. Whether McCleary and Droste-Shelah results can be fully recovered by our techniques remains to be seen.