Modelling concepts with affordance relations*

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Abstract

We aim to formalize the Gibsonian notion of an *affordance* relation and use it to explain (i) actions as labeled ternary relations arising from the interactions between actors and objects in a context (environment), and (ii) relational concepts as abstractions arising from actions.

1 The framework

We intend to model affordances and actions in the framework of property and information systems in the sense of Vakarelov (1998) and Pawlak (1982). A *property system* (*P-system*) is a structure $\langle U, V, f\rangle$, where U is a non-empty set whose elements are called *objects*, V is a set whose elements are called *properties*, and $f\colon U \to 2^V$ is a mapping called an *information function*; we do not require that $f(x) \neq \emptyset$. A statement $a \in f(u)$ can be interpreted as "Object u possesses property a". If U is finite, then a property system is definitionally equivalent to a formal dyadic context of Wille (1982), by observing that the function $f\colon U \to 2^V$ can be replaced by a relation $R_f \subseteq U \times V$, where $u\ R_f\ a$ if and only if $a \in f(u)$ which has the same informational content.[1]

If we think of a property system as describing possible states of an attribute—such as "color" or "language spoken"—we extend it by the definition of an aggregate structure: An *attribute system* (*A-system*) (Vakarelov, 1998) is a structure $\mathcal{S} \coloneqq \langle U, \Omega, \{V_a : a \in \Omega\}, f\rangle$ where

1. U is a non-empty set of objects,
2. Ω is a set of property labels or attributes, and V_a is a set of possible values of $a \in \Omega$,
3. $f\colon U \times \Omega \to \bigcup_{a\in\Omega} 2^{V_a}$ is a choice function, where $f(x, a) \subseteq V_a$. Equivalently, we may define $f : U \to \prod_{a\in\Omega} 2^{V_a}$.

So, if $a \in \Omega$ is a property label *weight*, then V_{weight} may be a set of rational numbers in some interval that can serve as numerical expressions of the weight of an object (e.g., in kilograms or pounds), or any value that makes sense. It could also be some aggregated value such as "low", "medium", "high" etc.

We call $\langle U, \Omega, \{V_a : a \in \Omega\}\rangle$ the *skeleton* of $\mathcal{S}$. The product $U \times \prod_{a\in\Omega} 2^{V_a}$ collects all possible vectors of value sets that can be associated with some element of U. An information function now picks one element from $\prod_{a\in\Omega} 2^{V_a}$ for each $x \in U$.

An element $x \in U$ is called *deterministic*, if $|f(x, a)| \leq 1$ for all $a \in \Omega$ or every projection of the vector attribute to x is either an empty set or a singleton subset of V_a. The set of

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[1] At this stage of our investigation we suppose that we have a correct description of the world, i.e. what we observe is true.

all deterministic elements of $\mathcal{S}$ is denoted by $D_{\mathcal{S}}$. The characterization stems from the fact that the role of the choice function is to narrow down the possibilities for the values of a with respect to the object x. If $|f(x,a)| = 1$, then we know the exact value of the property a for x, or if $|f(x,a)| = \emptyset$, then we know that x does not have this property at all. This is why we call a system with $|f(x,a)| \leqslant 1$ deterministic. If $|f(x,a)| \geq 2$ the set has different possibilities of interpretation, see (Düntsch et al., 2001). If U is finite and $U = D_{\mathcal{S}}$, then $\mathcal{S}$ is called an *information system* (in the sense of Pawlak, 1982).

2 Operationalizing affordances

A direction on operationalization of affordances was suggested in Düntsch et al. (2009):

> A formalization of affordance relations needs to provide crisp and fuzzy structures, mechanisms for spatial and temporal change, as well as contextual modeling.

The basic setup of an affordance relation consists of a set U of an agent's abilities, a set E of features of the environment, and a binary relation $R \subseteq U \times E$. Chemero (2003, p. 189) writes

> Affordances [...] are relations between the abilities of organisms and features of the environment. Affordances, that is, have the structure Affords–φ (feature,ability).

We expand this notion by regarding an affordance in a first step as a relation $\varphi \subseteq A \times O \times E$ between actors, objects and properties of the environment, where $\varphi(a, o, e)$ is interpreted as

> Entity o affords action $\texttt{Act}_{\varphi}$ to the actor (or perceiver, agent) a in the environment (context) e.

The initial notion of an affordance is quite coarse, and all three components require further description. Therefore, we extend the concept as follows: Suppose that for a set A of actors, a set O of entities or objects, and a set E of environmental factors we have deterministic information systems

$$\mathcal{I}_A = \left\langle A, \Omega_A, \{V_q^A : q \in \Omega_A\}, f_A : A \to \prod_{q\in\Omega_A} V_q^A \right\rangle,$$

$$\mathcal{I}_O = \left\langle O, \Omega_O, \{V_q^O : q \in \Omega_O\}, f_O : O \to \prod_{q\in\Omega_A} V_q^O \right\rangle,$$

$$\mathcal{I}_E = \left\langle E, \Omega_E, \{V_q^A : q \in \Omega_E\}, f_E : E \to \prod_{q\in\Omega_A} V_q^E \right\rangle.$$

Each of these information systems is interpreted as a description, respectively, of actors, entities, or the environment. We now define an *affordance* as a relation

$$\varphi \subseteq \{\langle a, f_A(a)\rangle : a \in A\} \times \{\langle o, f_O(o)\rangle : o \in O\} \times \{\langle e, f_E(e)\rangle : e \in E\}.$$

Thus, an affordance is a ternary relation that holds among actors with properties, objects with properties, and environments (contexts) with properties. See Figure 2 for a pictorial interpretation.

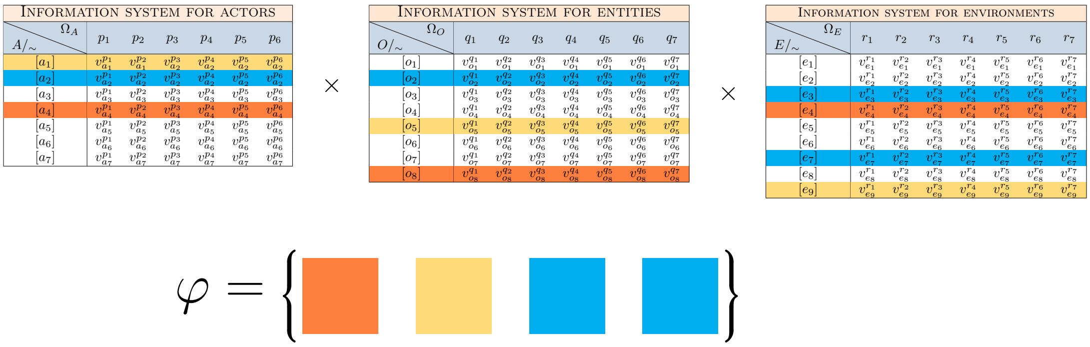

Figure 1: The triples of vectors of the same color constitute the affordance φ and its corresponding action $\texttt{Act}_\varphi$. We identify, respectively, actors, objects, and environments that cannot be distinguished by available properties.

3 Actions and concepts from affordances

A tuple in an affordance φ is interpreted as "action $\texttt{Act}_\varphi$ is afforded for actor a by entity o in the context e". A tuple $\langle \mathcal{I}_A, \mathcal{I}_O, \mathcal{I}_E, \varphi \rangle$ is called an *affordance structure*. For example, if actors are cleaning robots, objects are charging stations, and environments are interiors (e.g., offices or apartments), then we may think about, e.g., cyan triples from φ as

> charging stations of the type $[o_2]$, afford docking of robots of the type $[a_2]$ in interiors of types $[e_3]$ and $[e_7]$,

and similarly about triples of the two remaining colors.

Let us suppose that we also have a set of labels to tag affordances. These may be seen as finite strings of symbols over a finite alphabet Σ (that is, the Kleene closure Σ^* of Σ). Let us suppose that Σ is the standard Latin alphabet. Then, we may attribute the label '`dock`' to the affordance φ and thus obtain an action $\texttt{dock}_\varphi$ of *docking*. Thus actions are labeled affordances, more formally, they are elements of the set $\Sigma^* \times \texttt{Aff}$, where `Aff` is a set of affordances.

There is a similarity between $\texttt{dock}_\varphi$ and the action $\texttt{dock}_\psi$ of docking a ship in a shipyard. Clearly, we have different information systems composed of ships as actors, landing piers as objects, and shipyards as environments. Still further, we can see the similarity to the action $\texttt{dock}_\zeta$ where the information systems for the affordance ζ concern spaceships (actors), space stations (objects), and a low-gravity environment. We can now define concepts as abstractions from all actions of the type $\texttt{dock}_\delta$, where δ is an affordance. Speaking formally, the concept `DOCK` is a set of all actions of docking

$$\texttt{DOCK} \coloneqq \{\texttt{dock}_\varphi \mid \varphi \text{ is an affordance}\}\,.$$

We may say that concepts are abstractions from all those affordances to which we tend to attribute the same element of Σ^*.

The purpose of this presentation is to give details of our constructions and relate them to the other well-known formal theories of concepts, for example (Wille, 1982) and (Gärdenfors, 2000).

References

Chemero, A. (2003). An outline of a theory of affordances. *Ecological Psychology*, 15(2):181–195.

Düntsch, I., Gediga, G., and Lenarcic, A. (2009). Affordance relations. In Sakai, H., Chakraborty, M. K., Hassanien, A. E., Ślęzak, D., and Zhu, W., editors, *Proceedings of the Twelfth International Conference on Rough Sets, Fuzzy Sets, Data Mining & Granular Computing*, volume 5908 of *Lecture Notes in Computer Science*, pages 1–11. Springer Verlag.

Düntsch, I., Gediga, G., and Orłowska, E. (2001). Relational attribute systems. *International Journal of Human Computer Studies*, 55(3):293–309.

Düntsch, I., Gruszczyński, R., and Menchón, P. (2025). Reasoning with affordance relations. work in progress.

Gärdenfors, P. (2000). *Conceptual Spaces: The Geometry of Thought.* MIT Press, Cambridge, MA.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer and Information Sciences*, (11):341–356.

Vakarelov, D. (1998). Information systems, similarity relations and modal logics. In Orłowska, E., editor, *Incomplete information: Rough set analysis*, volume 13 of *Stud. Fuzziness Soft Comput.*, pages 492–550. Physica, Heidelberg. MR1647387.

Wille, R. (1982). Restructuring lattice theory: an approach based on hierarchies of concepts. In *Ordered sets (Banff, Alta., 1981)*, volume 83 of *NATO Adv. Study Inst. Ser. C: Math. Phys. Sci.*, pages 445–470. Reidel, Dordrecht-Boston, Mass. MR0661303.