Interpolation and Amalgamation in Many-Valued Logics

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Fuzzy logics have long enjoyed a fruitful symbiosis with algebraic methods. In this talk, I will discuss some recent successes of this symbiosis in the context of interpolation and amalgamation. We will see both how the advancement of algebraic techniques have enabled rapid progress on interpolation in fuzzy logics, as well as how the challenges posed by fuzzy logics have illuminated the path to new tools for the study of amalgamation in general algebraic systems.

The centerpiece of this discussion will be my recent classification with S. Santschi of varieties of BL-algebras with the amalgamation property, which yields also an exhaustive classification of axiomatic extensions of Hájek's basic fuzzy logic with the deductive interpolation property. I will discuss the main ideas and technical challenges of this classification, and what it can teach us about interpolation and amalgamation writ large.

[1] W. Fussner and G. Metcalfe, Transfer Theorems for Finitely Subdirectly Irreducible Algebras, J. Algebra 640:1-20 (2024).
[2] W. Fussner and S. Santschi, Amalgamation in Semilinear Residuated Lattices, manuscript (2024). Available at https://arxiv.org/abs/2407.21613.
[3] W. Fussner and S. Santschi. Interpolation in Hájek's Basic Logic, Ann. Pure. Appl. Logic 176(9), paper no. 103615 (2025).