A Logical Framework for Graded Deontic Reasoning — Introducing a New Research Project*

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1 Introduction

Logical reasoning about norms, obligations and permissions is important in many areas, from philosophy, legal considerations and social decisions to software development and AI. The development of the field is driven by various problems and puzzles of informal normative reasoning. For example, it is widely recognized that the adequate formalization of reasoning with defeasible deontic conditionals requires systems of modal logics that go beyond the mere addition of a standard (monadic) modal operator O (for 'it is obligatory') to classical logic. In particular, many intended applications require a dyadic modal operator where the semantics of $\mathsf{O}(\phi/\psi)$ ('ϕ should be the case if ψ holds') refers to a preference ordering for possible worlds (see [14]). Although this introduces *implicit (comparative) degrees of 'goodness'*, there is only very little research on logics with *explicitly graded deontic* propositions, so far.

Deontic propositions that allow for degrees of truth are anything but exceptional, but are probably part of the core of informal normative argumentation. The statements *'You should not kill innocent children'*, *'You should not lie'* and *'You should be polite'* are hardly appropriately categorized as *equally* true. Linguistic findings, confirm that typical deontic statements such as *'Peter should take care of Anna'*, *'Children are allowed to make noise'* or *'The place should be kept dry'* are gradable, as attested by the applicability of qualifiers such as *very much. probably* or *barely.* Moreover, the sentences that occur within a deontic modality are already typically gradable, as these examples show. Some deontic statements involve the comparison of gradations of applicability, as in *'The richer one is, the more one should donate to charity'*. For a detailed linguistic account of the gradability of *ought* and *should*, we refer to Section 8.13 of [11], where it is forcefully argued that

> In order to model this [documented linguistic] behaviour, we need to treat the basic form of *ought* as a scalar predicate associating propositions ϕ with "the degree to which ϕ ought to hold." [11], p. 249.

Another example that we will take up in Section 3.1 are questionnaires and opinion polls where people are asked to indicate on a certain scale (say 0 to 10) to what extent they agree with certain deontic statements, such as *'Taxes should be more progressive'* or *'Class sizes should be drastically reduced'*. Again, note that not only the deontic propositions themselves, but also the underlying non-modal propositions can reasonably be understood to admit degrees of truth.

Recognizing that it is appropriate and useful to consider graded deontic logics for logical models of normative reasoning does not entail that it is clear how such logics should look like.

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Indeed, there are many different parameters to consider when defining and examining such logics. It is not always clear which deontic (monadic or dyadic) modal operators are most appropriate. If we consider relevant comparative operators, further syntactic and semantic choices emerge. For example, one might want to model sentences of the form *'It is better that F than G'* or of the form *'Given that H holds, it is just as good that F as that G'*. When generalizing to a many-valued environment, one is confronted with a variety of options for the choice of a particular base logic. First of all, it is not clear with respect to which type of scale the relative or partial truth of deontic (and other) propositions should be comparable. The space of possible algebras for modeling the degrees of truth is large and diverse, as is known from the literature on mathematical fuzzy logics (see [2]). But even if one focuses on a particular fuzzy logic with the truth value set $[0, 1]$, e.g. Łukasiewicz logic or Gödel logic, to represent non-modal sentences, there are still many ways to extend many-valued semantics to deontic modal operators that scope over them.

We argue that the above challenge to research cannot be adequately met by examining particular many-valued deontic logics individually. Rather, one should attempt to provide a *logical framework* that can be instantiated in different ways to obtain concrete logical models of reasoning with graded norms, obligations, and permissions. Specifically, the project aims to develop a comprehensive semantic and syntactic proof-theoretic *toolbox* that allows a wide range of graded deontic logics and corresponding proof systems to be systematically defined, studied and compared.

2 A logical framework for graded deontic reasoning

Our aim is not just to develop new graded deontic logics, but rather to systematically explore the space of possibilities for defining such logics and to evaluate the resulting logics in terms of first principles of reasoning about relative goodness and degrees of desirability and of commitments. We briefly describe essential components for such a logical framework:

Goodness relations: Classical deontic logics with a dyadic obligation operator $\mathsf{O}(\phi/\psi)$ refer to a relation between possible worlds that orders these worlds with respect to their degree of goodness or desirability. Depending on the properties of this order and the way in which one refers to this order, different deontic logics emerge. For example, the following have been discussed in the literature for a relation $\succeq$ ('at least as good') on propositions (see, e.g., Lassiter [11]): $p \succeq q$ implies $\neg q \succeq \neg p$, $p \succeq q$ implies $p \wedge q \approx q$, or $p \succeq q$ implies $p \succeq p \vee q \succeq q$.

Relating degrees of goodness and degrees of truth: Goodness orders and corresponding degrees do not automatically translate into many-valued deontic propositions. Standard linguist approaches (see, e.g., [11]) suggest to use a context-dependent threshold value referring to the degree of goodness $\mu(\phi)$ of a proposition ϕ to decide whether an assertion of ϕ is acceptable (as true) or not. However, it also viable to identify $\mu(\phi)$ with a corresponding degree of truth to the proposition $\mathsf{O}(\phi)$ ('It should be the case that ϕ'). For the meaningfulness and fecundity of the latter strategy we refer to an analogous approach, due to Esteva, Godo and Hàjek [9], where 'probably' is interpreted as a graded modality by identifying the degree of truth of $\Pi(\phi)$ ('Probably ϕ') with the probability of an event corresponding to ϕ.

Choosing many-valued base logics via semantic games: There is a wide range of many-valued logics that may be used as underlying reasoning mechanism. In order to support the choice of suitable base logics we propose to employ semantic games (evaluation games) that characterize specific many-valued logics with respect to first principles about reasoning with graded propositions. The most prominent example of a semantic game of this type is Giles's game [8] characterizing Łukasiewicz logic by combining rules for the stepwise reduction of logically complex assertions to atomic assertions with an evaluation of the latter in terms of 'risk values'. Giles's game has been extended to other logics as well as to generalized (semi-fuzzy) quantifiers (see [4, 5] for an overview). We intend to extend Giles's game to include references to possible worlds as well as to goodness orders between worlds and propositions.

Incorporating uncertainty: Deontic (goodness) values are distinct from epistemic (uncertainty) values. However, as pointed out, e.g., by Lassiter [11], judgments about what should/ought to be the case are often not independent from expectations about the comparative likelihood of possible events. An important starting point for exploring the combination deontic and probabilistic reasoning is [3]. As the authors of [3] already indicate, one should consider generalizations of this approach to *dyadic* deontic logic, to other underlying base logics, and to various alternative measures of uncertainty.

Proof systems: We strive for soundness and completeness results for new logics emerging from the indicated framework. In particular, we claim that semantic games, as mentioned above, can be lifted to provability games using disjunctive states, which in turn correspond to analytic proof systems along the line of [6].

3 Two application areas

We decided to focus on two specific areas of applications within the project. Both are motivated by previous research of our group on that topics.

3.1 Judgment aggregation with graded deontic logics

Among the challenges for the assessment of fragmented and vague information is the systematic aggregation of opinions of many individuals. Classical Judgment Aggregation (JA) [10, 12, 13] poses the problem of finding a joint consistent collective judgment for a given agenda, modeled as a set of logically connected propositional formulas, based on individual bivalent (yes-no) judgments on the items in the agenda. Our attempts to generalize corresponding results to degree based deontic reasoning is based on two claims.

Claim A: In soliciting many opinions, one is often interested in what the individuals think *should* be done or *should* be the case.

Claim B: In soliciting opinions on—not only, but in particular—deontic propositions, it is useful and natural to allow for *degrees of assent/dissent*.

We will report on preliminary possibility and impossibility results for formal models of many-valued deontic judgment aggregation, extending recent results from [7].

3.2 Grading deontic situations

Puzzles for deontic logics typically deal with scenarios in which the described situation conflicts with a given norm set. For instance, in Forrester's paradox, the involved norm set is (1) "It is forbidden to murder (m)" and (2) "If one murders, one has to murder gently (g)". The puzzle arises in the situation, where m holds. To solve a deontic puzzle is to give a model that is consistent with the current situation and optimal with respect to the set of norms. In the case of Forrester's paradox, the solution is the situation $\{m, g\}$, which is consistent with in conflict with (1) but is consistent with m, satisfies (2), and is thus preferred over $\{m, \neg g\}$. In our context of graded deontic logic, the paradigmatic question arising from this discussion is: *How well* can a certain situation be resolved consistently with respect to a given set of norms?

We suggest to apply choice logics like QCL (see, e.g, [1]) to graded deontic scenarios to tackle this challenge. In this manner, new types of (graded) obligation operators arise. In particular, we will explore an alternative, game based semantics for the basic choice connective of QCL and for induced deontic operators, thus also providing an avenue for evaluating and expanding the applicability of our general framework.

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