Weighted logics and weighted automata

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Quantitative models and quantitative analysis in Computer Science are receiving increased attention. The goal of this talk is to investigate quantitative automata and quantitative logics. Weighted automata on finite words have already been investigated in seminal work of Schützenberger (1961). They consist of classical finite automata in which the transitions carry weights. These weights may model, e.g., the cost, the consumption of resources, or the reliability or probability of the successful execution of the transitions. This concept soon developed a flourishing theory, as is exemplified and presented in several books by Eilenberg, Salomaa-Soittola, Kuich-Salomaa, Berstel-Reutenauer, Sakarovitch, and the "Handbook of Weighted Automata". We investigate weighted automata and their relationship to weighted logics. For this, we present syntax and semantics of a quantitative logic; the semantics counts 'how often' a formula is true in a given word. Our main result, jointly with Paul Gastin, extending classical results of Büchi, Elgot and Trakhtenbrot (1961), shows that if the weights are taken from an arbitrary semiring, then weighted automata and a syntactically defined fragment of our weighted logic are expressively equivalent. A corresponding result holds for infinite words. Moreover, this extends to quantitative automata investigated by Henzinger et al. for modeling limit average-type or discounting behaviors e.g. of power plants. Finally, we consider Fagin's seminal result (1974) characterizing NP in terms of existential second-order logic; this started the field of descriptive complexity theory. In very recent work, jointly with Guillermo Badia, Carles Noguera and Erik Paul, we obtained a weighted version of Fagin's result.