Residuated lattices represented by twist-structures

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Residuated lattices arise in many contexts, particularly in algebraic logic, as they provide the algebraic semantics for substructural logics.

The class of residuated lattices is, in itself, a very broad class that includes a wide range of structures of different kinds, many of which are algebraic semantics of well-known and extensively studied propositional logics. Given this great diversity of algebras within the class, the systematic study of residuated lattices often employs constructions to obtain new structures from simpler or better-known ones. In this talk, we will focus on one such constructions, which has become well-known due to the variety of cases it covers: the twist-construction.

Although the first applications of twist-structures were used to obtain lattices with involution (Kalman in 1958), several other authors considered expansions with additional operations which induce new and interesting operations on the twist-structure. In particular, starting from a residuated lattice, the resulting construction yields a new one.

Our aim is to present a unified approach that offers a deeper insight into the classes of residuated lattices that admit a representation based on twist- structures. Our framework encompasses Nelson residuated lattices, Nelson paraconsistent residuated lattices, Kalman residuated lattices, among others. Moreover, we will show that with this approach we can also capture non-involutive twist-structures, such as Quasi-Nelson algebras and some of its variants, which have been recently introduced and studied mainly by U. Rivieccio. Our results enable comparisons among different twist-structures and provide some interesting new examples.

The ideas of presented are based on the works [1] and [2], done in collaboration with N. Galatos, M. Marcos and U. Rivieccio.

References

[1] M. Busaniche, N. Galatos and M. Marcos, Twist-structures and Nelson conuclei, Studia Logica **110** (2022), 949–987. https://doi.org/10.1007/s11225-022-09988-z.

[2] U. Rivieccio and M. Busaniche, Nelson conuclei and nuclei: the twist construction beyond involutivity, Studia Logica **112** (2024), 1123–1161. https://doi.org/10.1007/s11225-023-10088-9 .