When CalcVar meets Siena

Organized by
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January 19th – January 21th, 2026

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Abstracts

Geometric Transport Equation for currents: recent developments

Paolo Bonicatto

I will report on recent results concerning the Geometric Transport Equation for k-dimensional currents in $\mathbb{R}^n$. This equation generalises the classical continuity and transport equations to model the motion of geometric objects such as lines and surfaces. I will discuss well-posedness results for Lipschitz vector fields, highlighting a deep connection with the decomposability bundle of a measure introduced by Alberti and Marchese. This theory further extends to the time-dependent setting with minimal regularity in time of the vector field, thus offering a unified framework for the evolution of geometric data under non-smooth flows. If time allows, I will also outline a recent approach to the classical Frobenius' theorem via the transport of currents. The vanishing bracket condition is recast into transport identities that remain meaningful even when one of the vector fields is a normal 1-current and this perspective sheds light on some Alfvén-type statements in magnetohydrodynamics.

A "not so strange" term in the homogenisation of a problem with Robin boundary conditions

Giacomo Canevari
(joint work with K. Cherednichenko and A. Zarnescu)

We consider Laplace's equation in a periodically perforated domain, with Robin boundary conditions on the holes and a Robin coefficient inversely proportional to the total surface area of the holes. We show that, in a critical regime, the homogenised equation contains an additional zero-order term, which is defined in terms of a suitable eigenvalue problem and depends nonlinearly on the Robin coefficient. As the latter tends to infinity, the additional term converges to the capacitary "strange term" found by Cioranescu and Murat in the homogenisation of a problem with Dirichlet boundary conditions. This talk is based on joint work with K. Cherednichenko (University of Bath) and A. Zarnescu (BCAM, Bilbao, and "Simion Stoilow" Institute of the Romanian Academy).

An L^∞-variational problem for the Fractional Laplacian

Simone Carano
(joint work with R. Moser)

In this talk, we will discuss the minimisation of supremal functionals involving the fractional Laplace operator. We will establish the existence and uniqueness of global minimisers and, consequently, of absolute minimisers in the sense of Aronsson. In addition, we will show that the minimiser satisfies a corresponding fractional partial differential equation, which in turn yields a fractional eikonal property. This is based on joint work with Roger Moser (University of Bath).

A variational approach to topological singularities through Mumford-Shah type functionals

Lucia De Luca

(joint work with V. Crismale, N. Van Goethem and R. Scala)

We will present variational approaches to the analysis of topological singularities in the plane, starting from the - nowadays - classical Ginzburg-Landau (GL) model and core-radius (CR) approach. We will introduce a third approach inspired by the Mumford-Shah functional used in the context of image segmentation. Within our framework, the order parameter is an SBV map taking values in the unit sphere of the plane; the bulk energy is the squared L2 norm of the approximate gradient whereas the penalization term is given by the length of the jump set, scaled by a small parameter. After providing a notion of Jacobian determinant for SBV maps, we show that at any logarithmic scale our functional is "variationally equivalent" to the "standard" (CR) and (GL) models. Joint work with Vito Crismale, Nicolas Van Goethem and Riccardo Scala.

Direct Methods of the Calculus of Variations for superlinear free discontinuity problems

Matteo Focardi

(joint work with S. Conti and F. Iurlano)

Free-discontinuity energies whose bulk and surface densities exhibit superlinear growth, respectively for large gradients and small jump amplitudes, emerge in various fields, for instance in Imaging and in Fracture Mechanics. A distinctive feature of this kind of models is that the functionals are defined on $(G)SBV$ functions whose jump sets may have infinite measure. Establishing general lower semicontinuity and relaxation results in this setting requires new analytical techniques. In addition, variational approximation of superlinear energies via phase field models is key to the application of numerical methods. The talk is based on joint work with Sergio Conti (U. Bonn) and Flaviana Iurlano (U. Genova).

50 Years of Wave Equations with Time-Dependent Propagation Speed

Marina Ghisi
(joint work with M. Gobbino)

Ever since the classical work of De Giorgi, Colombini and Spagnolo in 1979, it has been known that the well-posedness of wave equations is extraordinarily sensitive to the time-regularity of the propagation speed. Over the decades that followed, a rich theory has emerged describing how continuity moduli, asymptotic stabilisation, and growth conditions on derivatives affect whether solutions exist, behave nicely, or exhibit derivative loss to some extent.

In this talk, we will present a new variational framework that brings together all of these classical results and pushes beyond them, covering for example propagation speeds in fractional Sobolev regimes. Perhaps most strikingly, we present an example showing that even severely irregular propagation speeds can still yield well-posed wave equations in Sobolev spaces.

The conclusion is provocative: there remains a huge gap between sufficient and necessary conditions, and we are far from knowing the true threshold between well-posedness and derivative loss.

(The talk is based on joint work with Massimo Gobbino).

A quantitative analysis of the staircasing phenomenon for the Perona-Malik equation

Massimo Gobbino
(joint work with N. Picenni)

The Perona-Malik functional is an integral functional whose Lagrangian is convex-concave with respect to the derivative, with a sublinear growth at infinity. As a consequence, its relaxation is identically zero. In order to mitigate this pathological behavior, several approximations have been proposed in the literature. Here we focus on the one obtained by adding a second-order term multiplied by a small parameter.

We investigate the asymptotic behavior of minima and minimizers as this parameter vanishes. In particular, we show that minimizers exhibit the so-called staircasing phenomenon, namely they develop a microstructure that resembles a piecewise constant function at suitable scales. We analyze the structure of this microstructure across different scales.

Our analysis relies on Gamma-convergence results for suitable rescaled functionals, blow-up techniques, and a characterization of local minimizers for the limit problem. This approach can be extended to more general models.

(Based on some joint papers with Nicola Picenni)

Lipschitz regularity of almost-minimizers in one-phase problems with generalized Orlicz growth

Chiara Leone

(joint work with G. Scilla, F. Solombrino and A. Verde)

Optimal local Lipschitz regularity for scalar almost minimizers of Alt-Caffarelli-type functionals

$$\mathcal{F}(v;\Omega) = \int_\Omega \varphi(x, |\nabla v(x)|) + \lambda \chi_{\{v>0\}}(x)\, dx,$$

with growth function φ a generalized Orlicz function, is established. The results presented in this talk have been obtained in collaboration with Giovanni Scilla (Napoli), Francesco Solombrino (Lecce), and Anna Verde (Napoli).

A rectifiability result for the 2d eikonal equation

Elio Marconi

(joint work with X. Lamy)

Given $\Omega \subset \mathbb{R}^2$ smooth domain and $\varepsilon > 0$, we consider the Aviles-Giga functionals

$$F_\varepsilon(u) = \int_\Omega \varepsilon |\nabla^2 u|^2 + \frac{1}{\varepsilon} |1 - |\nabla u|^2|^2 dx, \qquad u : \Omega \to \mathbb{R}.$$

The main conjecture about these functionals is that minimizers of F_ε under appropriate boundary conditions, tend to concentrate the energy on 1-dimensional rectifiable sets as $\varepsilon \to 0$. More precisely the conjectured Γ-limit of F_ε as $\varepsilon \to 0$ is

$$F_0(u) = \frac{1}{3} \int_{J_{\nabla u}} |u^+ - u^-|^3 d\mathcal{H}^1,$$

where $J_{\nabla u}$ is the 1-rectifiable set which detects the jump type discontinuity point of ∇u solving the eikonal equation $|\nabla u| = 1$. In this talk we review partial results about this conjecture and we show that, under suitable regularity assumptions on u, the best known candidate Γ-limit of F_ε as $\varepsilon \to 0$ has its energy concentrated on a 1-rectifiable set. This is a joint work with X. Lamy.

The isoperimetric inequality for the capillary energy outside convex sets

Massimiliano Morini

We study the isoperimetric problem for capillary hypersurfaces with a general contact angle outside arbitrary convex sets. We prove that the capillary energy of any surface supported on any such convex set is larger than that of a spherical cap enclosing the same volume and with the same contact angle at a flat support, and we characterize the equality cases. This provides a complete solution to the isoperimetric problem for capillary surfaces outside convex sets at arbitrary contact angles, generalizing the well-known Choe–Ghomi–Ritoré inequality, which corresponds to the case of an orthogonal contact.

Euler's elastica functional as a large mass limit of a nonlocal isoperimetric problem

Matteo Novaga

We consider the large mass limit of the non-local isoperimetric problem with a repulsive Yukawa potential in two space dimensions. In this limit, the non-local term concentrates on the boundary, resulting in the existence of a critical regime in which the perimeter and the non-local terms cancel each other out to leading order. We show that under appropriate scaling assumptions the next-order Γ-limit of the energy with respect to the L^1 convergence of the rescaled sets is given by a weighted sum of the perimeter and Euler's elastica functional, where the latter is understood via the lower-semicontinuous relaxation. As a consequence, we prove that in the considered regime the energy minimizers always exist and converge to either disks or annuli, depending on the relative strength of the elastic term.

Riesz fractional gradient functionals defined on partitions: nonlocal-to-local variational limits

Francesco Solombrino

(joint work with S. Almi, M. Friedrich and M. Caponi)

We address the asymptotics of functionals with linear growth depending on the Riesz s-fractional gradient on piecewise constant functions. We consider a general class of varying energy densities and, as s tends to 1, we characterize their local limiting functionals in the sense of Γ-convergence. This is a joint work with S. Almi (Napoli), M. Friedrich (Linz), and M. Caponi (L'Aquila).

Sharp-interface limit for non-isothermal and nonlocal Modica-Mortola functionals

Emanuele Tasso

(joint work with E. Davoli)

In this talk, we analyze a non-homogeneous and nonlocal variant of the Modica-Mortola diffuse model for phase transitions. Here the classical gradient penalization is replaced by a nonlocal singular perturbation and the double-well potential is space-dependent. Our main result is the identification of the sharp-interface limit as the width of the transition layers converges to zero. This is joint work with Elisa Davoli.

Multiscale analysis and homogenization of nonlocal thin films

Antonio Tribuzio
(joint work with N. Ansini)

In this talk, we will introduce a nonlocal, variational model for thin films. We consider family of functionals of convolution-type defined on a thin domain of thickness γ with the size of the most effective interactions between points being of order ε. After discussing the correct rescaling, we study the Γ-convergence of these energies as both parameters go to zero to a local integral functional defined on a lower dimensional domain. In the case of periodic (convex) homogenization, we can show that a "separation of scales" takes place. This is a work in progress in collaboration with Nadia Ansini.

The random fractional obstacle problem

Caterina Ida Zeppieri
(joint work with F. Deangelis and M. Focardi)

Nonlocal energies, such as fractional Sobolev seminorms, arise naturally in mathematical models involving long-range interactions. In this talk, we study minimizers of such energies that vanish on a collection of small balls with random centers and radii, leading to a bilateral (fractional) obstacle problem. In this talk I will present a homogenization result for such energies that holds under minimal assumptions on the distribution and size of the obstacles, which are generated by a stationary marked point process. In particular, the obstacles may overlap and form clusters with positive probability, giving rise to a complex microstructure. Our analysis identifies the limiting energy and shows how it reflects the underlying probability distribution of the obstacles. This is a joint work with Francesco Deangelis (University of Muenster) and Matteo Focardi (University of Florence).