Why CSP is easy

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The Constraint Satisfaction Problem (CSP) is the problem of deciding whether there is an assignment to a set of variables subject to some specified constraints. For over twenty years one of the main question was to classify constraint languages giving a tractable (solvable in polynomial time) CSP (like system of linear equations, 2-CNF, and so on). In 2017 the conjecture describing all tractable cases was independently proved by A. Bulatov and D. Zhuk.

In this talk I will argue that despite the fact that plenty of deep algebraic results and even theories appeared while studying the complexity of CSP, now we can say that Constraint Satisfaction Problem is actually easy.

First, the classification is very simple. The CSP is tractable if and only if the constraint language admits a weak near-unanimity (WNU) polymorphism, i.e there exists an operation w satisfying $w(y, x, x, \dots, x) = w(x, y, x, \dots, x) = \dots = w(x, x, \dots, x, y)$ and preserving the constraint language. Moreover, now we know that CSP can be solved by local methods if and only if the constraint language cannot express a system of linear equations, which is true if it admits a WNU polymorphism of every arity greater than 2.

Second, the most general tractable algorithm is just a smart combination of checking local consistency and solving of linear equations. Thus, instead of developing a completely new method for solving CSP, we learned how to use two well-known ideas for complicated constraint languages that combine several linear cases and several 2-CNF cases.

Third, the current proof does not really use any deep knowledge of universal algebra and tame congruence theory. Most facts can be proved just playing around with operations and relations (and a bit of absorption). Even though the current proof is still rather long and complicated, all key facts have simple and natural formulations.

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