The Tarski irredundant basis theorem and the generation of finite groups

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Denote by $d = d(G)$ and $m = m(G)$, respectively, the smallest and the largest cardinality of a minimal generating set of a finite group G. The Tarski irredundant basis theorem implies that for every k with $d(G) \leq k < m(G)$ there exist a minimal generating set $\omega = \{g_1, \ldots, g_k\}$, an index $i \in \{1, \ldots, k\}$ and x_1, x_2 in G such that $\tilde{\omega} = \{g_1, \ldots, g_{i-1}, x_1, x_2, g_{i+1}, \ldots, g_k\}$ is again a minimal generating set of G. In this case we say that $\tilde{\omega}$ is an immediate descendant of ω. There are several examples of minimal generating set of cardinality smaller than $m(G)$ which has no immediate descendant and so it appears an interesting problem to investigate under which conditions an immediate descendant exists. We may investigate the same problem considering the invariable generation (a subset S of a group G invariably generates G if, when each element of S is replaced by an arbitrary conjugate,the resulting set generates G). One can try to apply Tarski's argument to the invariable generation. This however will produce only a weak result, and the natural generalisation of Tarski's theorem to the invariable setting remains an open problem.

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