

# Sheaves on spectral spaces and the patch monad

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In this talk we will study sheaves over spectral spaces from the point of view of duality. A presheaf over a meet semilattice gives rise, via a construction introduced by Grothendieck, to a simple algebraic structure based on its elements, that is, the local sections of the presheaf. These are equipped with a semigroup structure given by the operation sometimes called restriction: given local sections  $s$  and  $t$ , the restriction of  $s$  to  $t$  is simply  $s$  restricted to the meet of the domains of  $s$  and  $t$ . The semigroups arising in this way are precisely the so-called normal bands studied early on by Kimura. Within this setting we will show that we can capture sheaves over spectral spaces, and that, in the special case of Boolean spaces these are in fact equivalent to skew Boolean algebras - a non-commutative variant of Boolean algebras introduced by Leech. In joint work with Andrej Bauer, Karin Cvetko-Vah, Sam van Gool, and Ganna Kudryavtseva, we showed in 2013 that skew distributive lattices are equivalent to sheaves on Priestley spaces. In order to capture this in the broader setting described here we have to study the monad on sheaves over spectral spaces induced by the identity map from the patch space of a spectral space to the spectral space itself. Indeed we show that skew distributive lattices are equivalent to algebras for this monad. This is recent joint work with Clemens Berger.

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